Course of Advanced Automation and Control

Exam for the students of the a.y. 2016/2017

June 13, 2017

Surname _____ Name _____

Part I - Optimization & Graphs (Prof. D.M. Raimondo)

1. The football club of Mr. Grecchi is looking for a new striker for the next three years. The talent scouts have identified two extremely good players: Oliver Hutton and Mark Lenders. Table 1 reports the price for buying and/or selling players. The price is different depending on when the players are bought/sold. The club buys/sells players at the beginning of each year. This means that if a player is bought at Y1 and sold at Y2, he will play Y1 only.

	Buying cost Y1	Buying/selling cost Y2	Buying/selling cost Y3
H. Hutton	10^8 *Euro*	$9.5 * 10^7$ *Euro*	$8 * 10^7$ *Euro*
M. Lenders	$9 * 10^7$ *Euro*	$8.5 * 10^7$ *Euro*	$8 * 10^7$ *Euro*

Table 1

Hint: in order to model the buying/selling strategy, one can use binary variables for buying (one per year for each player) and selling (one per Y2 and one per Y3 for each player since it is not possible to sell a player at Y1). Note that:

- a striker can be bought and sold only once.
- a striker cannot be hired and sold the same year.
- a striker can be sold only if he was previously hired.
- only one striker is hired per year.
- the striker can be different over the years.

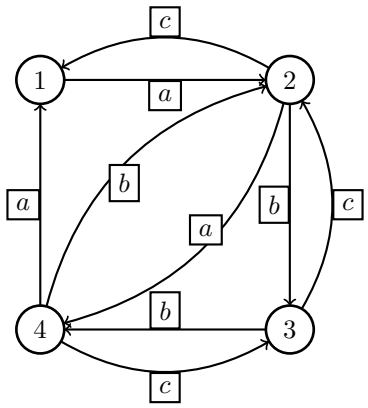
Besides the fixed cost of buying/selling a player, there is the salary, which varies depending on the year

	salary Y1	salary Y2	salary Y3
H. Hutton	10^7 *Euro*	10^7 *Euro*	10^7 *Euro*
M. Lenders	10^7 *Euro*	$9 * 10^6$ *Euro*	$8 * 10^6$ *Euro*

Clearly a player gets paid only if bought and not sold yet. The salary cost can be computed every year taking into account the difference between the binary variables. For example a player will get paid for Y2 an amount which is equal to $salary * (binary\ buy\ Y1 + binary\ buy\ Y2 - binary\ sell\ Y2)$. The same can be done for all years. The next table reports the average number of goals each player can score per year.

	avg. goals Y1	avg. goals Y2	avg. goals Y3
H. Hutton	33	32	30
M. Lenders	30	29	28

The objective of Mr. Grecchi is to minimize the net costs while guaranteeing an average number of goals greater or equal to 90 over the next three years. Please formulate the problem as a mixed integer program.



2. Consider the automaton in the figure above.

   where $C = \{a, b, c\}$ is the set of control values and $S = \{1, 2, 3, 4\}$ is the set of state values. Let the intermediate cost $g(x, u)$ and the terminal cost $g_2(x)$ be given by

$g(x,u)$	a	b	c
1	2	-	-
2	5	2	1
3	-	4	0
4	2	1	1

$$g_3(x) = \begin{cases} 1 & \text{if } x = 1 \\ 2 & \text{if } x = 2 \\ 3 & \text{if } x = 3 \\ 4 & \text{if } x = 4 \end{cases}$$

   2.1 Solve the optimal control problem

$$J(x_0) = \min_{u_0, u_1, u_2} g_3(x_3) + \sum_{k=0}^{2} g(x_k, u_k)$$

   using dynamic programming.

   2.2 Compute an optimal control sequence for $x_0 = 3$ and compute the optimal cost value.

3. Please solve the following MILP problem using the branch and bound algorithm

$$\begin{array}{ll} \max_{x_1, \delta_1, \delta_2} & -x_1 + 0.5\delta_1 + 4\delta_2 \\ & 3x_1 + 2\delta_2 \qquad\qquad \leq 1.5 \\ & \delta_1, \delta_2 \in \{0, 1\} \\ & x_1 \geq 0 \end{array}$$